

THEORY OF INCLUSIVE SCATTERING OF POLARIZED ELECTRONS BY POLARIZED ^3He AND THE NEUTRON FORM FACTORS¹

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ABSTRACT

The theory of inclusive lepton scattering of polarized leptons by polarized $J = 1/2$ hadrons is presented and the origin of different expressions for the polarized nuclear response function appearing in the literature is explained. The sensitivity of the longitudinal asymmetry upon the neutron form factors is investigated.

1. Introduction

The scattering of polarized electrons by polarized targets represents a valuable tool for investigating nucleon and nuclear properties in great detail¹. In particular quasi-elastic (qe) inclusive experiments involving polarized ^3He ^{2,3} are viewed as possible source of information on the neutron form factors; as a matter of fact, if only the main component (S-wave) of the three-body wave function is considered, the proton contribution to the asymmetry largely cancels out. The proton contribution, arising from S' and D-waves, has been studied in Ref.[4] within the closure approximation, i.e., by describing nuclear effects through spin-dependent momentum distributions. Adopting the general formalism of Ref.[4], the effects of nucleon binding has been analysed in Ref.[5], where the concept of the spin dependent spectral function has been introduced and applied to the calculation of the ^3He asymmetry. The effects of binding has also been recently considered in Ref.[6], where a new expression for the polarized nuclear structure functions has been obtained. In that paper, moreover, doubts have been raised as to the possibility of obtaining reliable information on the neutron form factors by the measurement of the inclusive asymmetry.

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Since a clear explanation about the origin of the differences between the expression of the polarized structure functions used in Refs.[4,5] and the one obtained in Ref.[6] is lacking in the literature, the aim of this paper is first to present a comprehensive derivation of the inclusive cross section, in order to clarify the origin of the above mentioned differences, and, second, to show that, provided a proper kinematics is chosen, the qe asymmetry can be made very sensitive to neutron properties, and in particular to the neutron electric form factors.

2. The hadronic tensor and the polarized structure functions

The inclusive cross section describing the scattering of a longitudinally polarized lepton of helicity $h = \pm 1$ by a polarized hadron of spin $J = 1/2$, is given in one photon exchange approximation by ¹

$$\begin{aligned} \frac{d^2\sigma(h)}{d\Omega_2 d\nu} &\equiv \sigma_2(\nu, Q^2, \vec{S}_A, h) = \frac{4\alpha^2}{Q^4} \frac{\epsilon_2}{\epsilon_1} m^2 L^{\mu\nu} W_{\mu\nu} = \\ &= \frac{4\alpha^2}{Q^4} \frac{\epsilon_2}{\epsilon_1} m^2 [L_s^{\mu\nu} W_{\mu\nu}^s + L_a^{\mu\nu} W_{\mu\nu}^a] \end{aligned} \quad (1)$$

where the symmetric (s) and antisymmetric (a) leptonic tensors are

$$L_s^{\mu\nu} = -(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}) \frac{Q^2}{4m^2} + \frac{1}{m^2} (k_1^\mu - \frac{q^\mu}{2}) (k_1^\nu - \frac{q^\nu}{2}) \quad (2)$$

$$L_a^{\mu\nu} = i h \epsilon^{\mu\nu\rho\sigma} \frac{q_\rho k_{1\sigma}}{2m^2} \quad (3)$$

and the corresponding hadronic tensors are

$$W_{\mu\nu}^s = -(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}) W_1^A + (P_{A\mu} + \frac{P_A \cdot q}{Q^2} q_\mu) (P_{A\nu} + \frac{P_A \cdot q}{Q^2} q_\nu) \frac{W_2^A}{M_A^2} \quad (4)$$

$$W_{\mu\nu}^a = i \epsilon_{\mu\nu\rho\sigma} q^\rho V^\sigma \quad (5)$$

The pseudovector V^σ appearing in Eq.(5) can be expressed as follows

$$V^\sigma \equiv S_A^\sigma \frac{G_1^A}{M_A} + (P_A \cdot q S_A^\sigma - S_A \cdot q P_A^\sigma) \frac{G_2^A}{M_A^3} \quad (6)$$

In the above equations, the index A denotes the number of nucleons composing the target, $k_{1(2)}^\mu \equiv (\epsilon_{1(2)}, \vec{k}_{1(2)})$ and $P_A^\mu \equiv (M_A, 0)$ are electron and target four-momenta, $q^\mu \equiv (\nu, \vec{q})$ is the four-momentum transfer, $Q^2 = -q^2$, $g_{\mu\nu}$ is the symmetric metric tensor, $\epsilon_{\mu\nu\rho\sigma}$ the fully antisymmetric tensor, and S_A^μ the polarization four-vector (in the rest frame $S_A^\mu \equiv (0, \vec{S}_A)$).

Although both the symmetric, $W_{\mu\nu}^s$, and the antisymmetric, $W_{\mu\nu}^a$, parts of the hadronic tensor are involved in the polarized scattering, in what follows we will focus on the antisymmetric one, since it contains the relevant physical quantities we want to investigate. To this end the following remarks are in order: i) as usual, the general form of $W_{\mu\nu}^a$, (Eq.(5)), can be obtained only by using invariance principles (Lorentz, gauge, parity and time reversal invariance); ii) since the antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$ in Eq.(5) cancels out the contribution to $W_{\mu\nu}^a$ arising from any term proportional to q^μ ($\epsilon_{\mu\nu\alpha\beta} q^\alpha q^\beta = 0$), terms of this kind, which in principle could appear in the definition of V^σ , were not included in Eq.(6). The relevance of this last comment will be clear later on, when the method for obtaining the polarized structure functions in the framework of PWIA will be discussed.

In total analogy with the case of the unpolarized structure functions which are determined by the symmetric part of the hadronic tensor, the polarized structure functions G_1^A and G_2^A have to be obtained by expressing them in terms of the components of $W_{\mu\nu}^a$. This can be accomplished by "inverting" Eq.(5). In fact one has

$$\tilde{V}^\sigma \equiv V^\sigma + \frac{q^\sigma}{Q^2} (V \cdot q) = i \frac{1}{2 Q^2} \epsilon^{\sigma\alpha\mu\nu} q_\alpha W_{\mu\nu}^a \quad (7)$$

where the four-vector \tilde{V}^σ is orthogonal to q_σ , i.e. $\tilde{V} \cdot q = 0$. Thus, working in the rest frame of the target, assuming the z-axis along the momentum transfer ($\hat{q} \equiv \hat{u}_z$), and using Eq.(6), the following expressions for the polarized structure functions G_1^A and G_2^A are obtained (cf. Ref.[6])

$$\frac{G_1^A}{M_A} = -i \left(\frac{Q^2}{|\vec{q}|^3} \frac{W_{02}^a}{S_{Ax}} + \frac{\nu}{|\vec{q}|^2} \frac{W_{12}^a}{S_{Az}} \right) \quad (8)$$

$$\frac{G_2^A}{M_A^2} = -i \frac{1}{|\vec{q}|^2} \left(\frac{\nu}{|\vec{q}|} \frac{W_{02}^a}{S_{Ax}} - \frac{W_{12}^a}{S_{Az}} \right) \quad (9)$$

It should be stressed that, in line with the above remark ii), Eqs.(8) and (9) are not affected, because of Eq.(5), by any arbitrary term proportional to q^σ which could be added to V^σ given by Eq.(6). At this point, it should be pointed out that in Ref.[4] $G_{1(2)}^A$ have been obtained by another procedure, namely by expressing them in terms of the components of the pseudovector V^σ , given by Eq.(6). One obtains in this case

$$\frac{G_1^A}{M_A} = -\frac{(V \cdot q)}{|\vec{q}| S_{Az}} \quad (10)$$

$$\frac{G_2^A}{M_A^2} = \frac{V_0}{|\vec{q}| S_{Az}} \quad (11)$$

Given the form (6) for V^σ , Eqs.(8) and (9) are totally equivalent to Eqs.(10) and (11). However, such an equivalence will not hold if a term proportional to q^μ is explicitly

added to the r.h.s. of Eq.(6), since Eqs.(8) and (9) will be not affected by the added term, whereas Eqs.(10) and (11) will be; therefore G_1^A and G_2^A obtained from Eqs.(10) and (11) will be not correct. This remark will be very relevant in the discussion of the evaluation of G_1^A and G_2^A in Plane Wave Impulse Approximation (PWIA) that will be presented in the next Section. To sum up, unlike the unpolarized case, two different procedures have been followed to obtain the polarized structure functions $G_{1(2)}^A$; they lead to Eqs.(8) and (9) and Eqs.(10) and (11), respectively; however the latters are correct only in so far as V^σ is a linear combination of only S_A^σ and P_A^σ and terms proportional to q^σ are absent in the definition of V^σ . We will call the correct prescription leading to Eqs.(8) and (9) prescription I (corresponding to the prescription A of Ref.[6]) and that leading to Eqs.(10) and (11) prescription II (corresponding to the prescription C of Ref.[6] and originally proposed in Ref.[4]).

3. The polarized structure functions in PWIA

The equations given in Sect.2 are general ones, relying on the one photon exchange approximation. When comparing with experimental data, one has to adopt some models for the nuclear structure functions. All papers so far published^{4,5,6} use the PWIA. Within such an approximation, one can obtain the following expression for the antisymmetric hadronic tensor

$$w_{\mu\nu}^a = i\epsilon_{\mu\nu\alpha\beta} q^\alpha R^\beta \quad (12)$$

where the four-pseudovector R^β is given by

$$R^\beta = \sum_{i=p,n} \left[\frac{\tilde{G}_1^i(Q^2)}{M} \langle S^\beta \rangle_i + \frac{\tilde{G}_2^i(Q^2)}{M^3} q_\alpha (\langle p^\alpha S^\beta \rangle_i - \langle p^\beta S^\alpha \rangle_i) \right] \quad (13)$$

In Eq.(13) $p^\alpha \equiv (\sqrt{M^2 + |\vec{p}|^2}, \vec{p})$ is the on-shell nucleon momentum, $\tilde{G}_1^{p(n)}(Q^2)$ and $\tilde{G}_2^{p(n)}(Q^2)$ are the proton (neutron) spin-dependent form factors, related to the Sachs form factors by the following equations⁴

$$\tilde{G}_1^{p(n)}(Q^2) = -\frac{G_M^{p(n)}}{2} \frac{(G_E^{p(n)} + \tau G_M^{p(n)})}{(1 + \tau)} \quad (14)$$

$$\tilde{G}_2^{p(n)}(Q^2) = \frac{G_M^{p(n)}}{4} \frac{(G_M^{p(n)} - G_E^{p(n)})}{(1 + \tau)} \quad (15)$$

with $\tau = Q^2/(4M^2)$, and

$$\begin{aligned} \langle (p^\alpha) S^\beta \rangle_{p(n)} &= \int dE_{(A-1)}^f \int d\vec{p} \frac{M^2}{E_p E_{p+q}} (p^\alpha) \sum_{l=1,3} f_{\mathcal{M},l}^{p(n)}(\vec{p}, E) S_l^\beta \\ &\quad \delta(\nu + M_A - \sqrt{(M_{A-1} + E_{A-1}^f)^2 + |\vec{p}|^2} - E_{p+q}) \end{aligned} \quad (16)$$

In Eq.(16) $E_{p+q} = \sqrt{M^2 + (\vec{p} + \vec{q})^2}$, $E = E_{A-1}^f - E_A$ is the nucleon removal energy, and $S_l^\beta \equiv (\hat{u}_l \cdot \vec{p}/M, \hat{u}_l + \vec{p} \cdot \hat{u}_l / (M(E_p + M)))$ is the polarization of a moving nucleon, which in its rest frame has the spin along the direction of the l -axis, whose versor is \hat{u}_l ($l = x, y, z$). Eqs.(12) and (16) are a generalization of the expressions of Ref.[4] to the case where both the nucleon momentum and energy distributions are considered.

In Eq.(16) the three-dimensional pseudovector $\vec{f}_{\mathcal{M}}^{p(n)}(\vec{p}, E)$ describes the nuclear structure and is defined as follows

$$\vec{f}_{\mathcal{M}}^{p(n)}(\vec{p}, E) = \text{Tr} \left(\hat{\mathbf{P}}_{\mathcal{M}}^{p(n)}(\vec{p}, E) \vec{\sigma} \right) \quad (17)$$

where the 2x2 matrix $\hat{\mathbf{P}}_{\mathcal{M}}^{p(n)}(\vec{p}, E)$ is the spin dependent spectral function of a nucleon inside a nucleus with polarization \vec{S}_A oriented, in general, in a direction different from the z-axis, and \mathcal{M} is the component of the total angular momentum along \vec{S}_A . The elements of the matrix $\hat{\mathbf{P}}_{\mathcal{M}}^{p(n)}(\vec{p}, E)$ are given by

$$P_{\sigma, \sigma', \mathcal{M}}^N(\vec{p}, E) = \sum_{f_{A-1}} N \langle \vec{p}, \sigma; \psi_{A-1}^f | \psi_{J\mathcal{M}} \rangle \langle \psi_{J\mathcal{M}} | \psi_{A-1}^f; \vec{p}, \sigma' \rangle_N \delta(E - E_{A-1}^f + E_A) \quad (18)$$

where $|\psi_{J\mathcal{M}}\rangle$ is the ground state of the target nucleus polarized along \vec{S}_A , $|\psi_{A-1}^f\rangle$ is an eigenstate of the (A-1) nucleon system, $|\vec{p}, \sigma\rangle_N$ is the plane wave for the nucleon $N \equiv p(n)$ with the spin along the z-axis equal to σ . It should be pointed out that, for a $J = 1/2$ nucleus, the trace of $\hat{\mathbf{P}}_{\mathcal{M}}^{p(n)}(\vec{p}, E)$ yields the usual unpolarized spectral function⁷. The spin dependent spectral function of ^3He ⁵ has been first obtained from the overlap integrals, (Eq.(18)), corresponding to a variational wave function for the Reid soft-core interaction. The same quantity has been calculated in Ref.[6], but using a Faddeev wave function and the Paris potential.

Since $\vec{f}_{\mathcal{M}}^{p(n)}(\vec{p}, E)$ is a pseudovector, it is a linear combination of the pseudovectors at our disposal, viz. \vec{S}_A and \hat{p} ($\hat{p} \cdot \vec{S}_A$), and therefore it can be put in the following form, where any angular dependence is explicitly given,

$$\vec{f}_{\mathcal{M}}^{p(n)}(\vec{p}, E) = \vec{S}_A B_1^{p(n)}(p, E) + \hat{p} (\hat{p} \cdot \vec{S}_A) B_2^{p(n)}(p, E) \quad (19)$$

with $p \equiv |\vec{p}|$. The relations between $B_1^{p(n)}$, $B_2^{p(n)}$ and the quantities $P_{\parallel}^{p(n)}(p, E, \alpha)$ and $P_{\perp}^{p(n)}(p, E, \alpha)$ used in our previous paper⁵ can be easily found from Eqs.(17), (18) and (19) by assuming $\vec{S}_A \equiv \hat{q} \equiv (0, 0, 1)$, viz.

$$P_{\parallel}^{p(n)}(p, E, \alpha) = B_1^{p(n)}(p, E) + B_2^{p(n)}(p, E) \cos^2 \alpha \quad (20)$$

$$P_{\perp}^{p(n)}(p, E, \alpha) = B_2^{p(n)}(p, E) \cos \alpha \sin \alpha \quad (21)$$

with $\cos\alpha = \hat{p} \cdot \hat{q}$.

The evaluation of G_A^1 and G_A^2 in the framework of PWIA can be carried out by substituting in Eqs.(8) and (9) the elements of the PWIA hadronic tensor $w_{\mu\nu}^a$ obtained from Eqs.(12), (13) and (16), and by using the functions $B_{1(2)}^{p(n)}$ obtained from Eqs.(20) and (21). One gets

$$\frac{G_A^1(Q^2, \nu)}{M_A} = 2\pi \sum_{i=p,n} \int_{E_{min}}^{E_{max}(Q^2, \nu)} dE \int_{p_{min}(Q^2, \nu, E)}^{p_{max}(Q^2, \nu, E)} \frac{p}{|\vec{q}|E_p} dp \left\{ \tilde{G}_1^{(i)}(Q^2) \left[M P_{\parallel}^{(i)}(p, E, \alpha) + \right. \right. \\ \left. \left. - p \left(\frac{\nu}{|\vec{q}|} - \frac{p \cos\alpha}{M + E_p} \right) \mathcal{P}^{(i)}(p, E, \alpha) \right] - \frac{Q^2}{|\vec{q}|^2} \mathcal{L} \right\} \quad (22)$$

$$\frac{G_A^2(Q^2, \nu)}{M_A^2} = 2\pi \sum_{i=p,n} \int_{E_{min}}^{E_{max}(Q^2, \nu)} dE \int_{p_{min}(Q^2, \nu, E)}^{p_{max}(Q^2, \nu, E)} \frac{p}{|\vec{q}|E_p} dp \left\{ \left[\tilde{G}_1^{(i)}(Q^2) \frac{p}{|\vec{q}|} \mathcal{P}^{(i)}(p, E, \alpha) + \right. \right. \\ \left. \left. + \frac{\tilde{G}_2^{(i)}(Q^2)}{M} \left(E_p P_{\parallel}^{(i)}(p, E, \alpha) - \frac{p^2 \cos\alpha}{M + E_p} \mathcal{P}^{(i)}(p, E, \alpha) \right) \right] - \frac{\nu}{|\vec{q}|^2} \mathcal{L} \right\} \quad (23)$$

with

$$\mathcal{L} = \left[\tilde{G}_1^{(i)}(Q^2) \mathcal{H}_1^i + |\vec{q}| \frac{\tilde{G}_2^{(i)}(Q^2)}{M} \mathcal{H}_2^i \right] \quad (24)$$

$$\mathcal{H}_1^i = \frac{1}{2} \frac{(3 \cos^2\alpha - 1)}{\cos\alpha} \left[\frac{p^2}{M + E_p} \mathcal{P}^{(i)}(p, E, \alpha) + M \frac{P_{\perp}^{(i)}(p, E, \alpha)}{\sin\alpha} \right] \quad (25)$$

$$\mathcal{H}_2^i = p \left[\mathcal{P}^{(i)}(p, E, \alpha) - \frac{P_{\perp}^{(i)}(p, E, \alpha)}{\sin\alpha} \right] + \\ - \frac{\nu}{2|\vec{q}|} \frac{(3 \cos^2\alpha - 1)}{\cos\alpha} \left[\frac{p^2}{M + E_p} \mathcal{P}^{(i)}(p, E, \alpha) - E_p \frac{P_{\perp}^{(i)}(p, E, \alpha)}{\sin\alpha} \right] \quad (26)$$

and $\mathcal{P}^{(i)}(p, E, \alpha) = \cos\alpha P_{\parallel}^{(i)}(p, E, \alpha) + \sin\alpha P_{\perp}^{(i)}(p, E, \alpha)$. In Eqs.(22) and (23) the integration limits and $\cos\alpha$ are determined, as usual, through the energy conservation⁸. The polarized structure functions $G_{1(2)}^A$, given by Eqs.(22) and (23), coincide with the ones corresponding to the extraction scheme (A) of Ref. [6], once the off-shell effects are neglected, and $P_{\parallel(\perp)}^{(i)}(p, E, \alpha)$ are expressed in terms of the scalar functions f_1 and f_2 introduced in Ref.[6].

It should be pointed out that $G_{1(2)}^A$, obtained in Ref.[5], differ from Eqs.(22) and (23) in that they do not contain the term \mathcal{L} . The origin of such a difference is that in

Ref.[5] we followed the procedure of Ref.[4], according to which the polarized structure functions are obtained from Eqs.(10) and (11) by directly substituting the four-vector V^σ with the four-vector R^σ . As already explained in Sect.2, such a procedure will lead to Eqs.(22) and (23) only if the functional dependence of the two four-vectors is the same, i.e. if R^σ is a linear combination of only S_A^σ and P_A^σ ; it turns out by an explicit evaluation of Eq.(13)⁹ that this is not the case, for R^σ contains a term proportional to q^σ , which is washed out by $\epsilon_{\mu\nu\alpha\sigma}$ when the hadronic tensor is evaluated.

4. The asymmetry in the quasi-elastic region

The contraction of the two tensors in Eq.(1) yields

$$\frac{d^2\sigma(h)}{d\Omega_2 d\nu} = \Sigma + h \Delta \quad (27)$$

where

$$\Sigma = \sigma_{Mott} \left[W_2^A(Q^2, \nu) + 2 \tan^2 \frac{\theta_e}{2} W_1^A(Q^2, \nu) \right] \quad (28)$$

$$\Delta = \sigma_{Mott} 2 \tan^2 \frac{\theta_e}{2} \left[\frac{G_1^A(Q^2, \nu)}{M_A} (\vec{k}_1 + \vec{k}_2) + 2 \frac{G_2^A(Q^2, \nu)}{M_A^2} (\epsilon_1 \vec{k}_2 - \epsilon_2 \vec{k}_1) \right] \cdot \vec{S}_A \quad (29)$$

In what follows, the target polarization vector \vec{S}_A is supposed to lie within the scattering plane formed by \vec{k}_1 and \vec{k}_2 .

Two possible kinematical conditions can be considered:

β - *kinematics*. The target polarization angle is measured with respect to the direction of the incident electron, i.e. $\cos\beta = \vec{S}_A \cdot \vec{k}_1 / |\vec{k}_1|$, (this is the more suitable choice from the experimental point of view). In this case, one gets

$$\begin{aligned} \Delta \equiv \Delta_\beta = \sigma_{Mott} 2 \tan^2 \frac{\theta_e}{2} & \left\{ \frac{G_1^A(Q^2, \nu)}{M_A} [\epsilon_1 \cos\beta + \epsilon_2 \cos(\theta_e - \beta)] + \right. \\ & \left. - 2 \frac{G_2^A(Q^2, \nu)}{M_A^2} \epsilon_1 \epsilon_2 [\cos\beta - \cos(\theta_e - \beta)] \right\} \end{aligned} \quad (30)$$

θ^* - *kinematics*. The target polarization angle is measured with respect to the direction of the momentum transfer, i.e. $\cos\theta^* = \vec{S}_A \cdot \vec{q} / |\vec{q}|$. Then one can write

$$\begin{aligned} \Delta \equiv \Delta_{\theta^*} = -\sigma_{Mott} \tan \frac{\theta_e}{2} & \left\{ \cos\theta^* R_{T'}^A(Q^2, \nu) \left[\frac{Q^2}{|\vec{q}|^2} + \tan^2 \frac{\theta_e}{2} \right]^{1/2} + \right. \\ & \left. - \frac{Q^2}{|\vec{q}|^2 \sqrt{2}} \sin\theta^* R_{TL'}^A(Q^2, \nu) \right\} \end{aligned} \quad (31)$$

where

$$R_{T'}^A(Q^2, \nu) = -2 \left(\frac{G_1^A(Q^2, \nu)}{M_A} \nu - Q^2 \frac{G_2^A(Q^2, \nu)}{M_A^2} \right) = i 2 \frac{W_{12}^a}{S_{Az}} \quad (32)$$

$$R_{TL'}^A(Q^2, \nu) = 2 \sqrt{2} |\vec{q}| \left(\frac{G_1^A(Q^2, \nu)}{M_A} + \nu \frac{G_2^A(Q^2, \nu)}{M_A^2} \right) = -i 2 \sqrt{2} \frac{W_{02}^a}{S_{Ax}} \quad (33)$$

In principle the θ^* - kinematics is very appealing, since by performing experiments at $\theta^* = 0$ and 90° one can disentangle $R_{T'}^A$ and $R_{TL'}^A$, which, at the top of the qe peak, are proportional to $(G_M^n)^2$ and $G_E^n G_M^n$, respectively, provided the proton contribution can be disregarded^{2,3}.

Experimentally one measures the asymmetry

$$A = \frac{\sigma_2(\nu, Q^2, \vec{S}_A, +1) - \sigma_2(\nu, Q^2, \vec{S}_A, -1)}{\sigma_2(\nu, Q^2, \vec{S}_A, +1) + \sigma_2(\nu, Q^2, \vec{S}_A, -1)} = \frac{\Delta}{\Sigma} \quad (34)$$

If the naive model of ^3He holds, this quantity is in principle very sensitive to the neutron properties, since the numerator should be essentially given by the neutron with its spin aligned along \vec{S}_A . With this simple picture in mind, let us consider the comparison between our results based on Eqs(22) and (23), with the experimental data obtained at MIT-Bates^{2,3}.

In Fig. 1 the asymmetry corresponding to $\epsilon_1 = 574 \text{ MeV}$ and $\theta_e = 44^\circ$, measured by the MIT-Caltech collaboration² is shown. The experimental data were obtained in a large interval of the energy transfer after averaging over three different values of the β angle ($\beta = 44.5^\circ, 51.5^\circ, 135.5^\circ$ with the corresponding azimuthal angles being: $\phi = 180^\circ, 180^\circ, 0^\circ$). It is worth noting that in these kinematical conditions one has $\theta^* \approx 90^\circ$ only at the top of the qe peak, and therefore only there the measured asymmetry reduces to $R_{TL'}$. In the figure the neutron (dotted line) and proton (dashed line) contributions are separately shown, and the relevance of the proton contribution can be seen particularly at the top of the qe peak ($A_{qe}^{exp} \propto R_{TL'}^{exp}$), where a comparison with the experimental values (obtained from a further averaging over an interval of the energy transfer of about 100 MeV) yields

$$\begin{aligned} A_{qe}^{exp} &= 2.41 \pm 1.29 \pm 0.51 \% \text{ MIT - Caltech}^2 \\ A_{qe}^{exp} &= 1.75 \pm 1.20 \pm 0.31 \% \text{ MIT - Harvard}^3 \\ A^{th} &= 3.74 \% \\ A_p^{th} &= 2.20 \% \end{aligned}$$

In Fig.2 the theoretical asymmetry, averaged over the same values of the polarization angle of the previous case, is shown in correspondence with $\epsilon_1 = 574 \text{ MeV}$

and $\theta_e = 51.1^\circ$. Such a kinematics was chosen^{2,3} with the aim of extracting $R_{T'}$ at the qe peak. Only one experimental point has been obtained for the averaged asymmetry around the top of the qe peak, where $\theta^* \approx 0^\circ$ ($A_{qe}^{exp} \propto R_{T'}^{exp}$). The comparison between the experimental results and our calculation is as follows

$$\begin{aligned} A_{qe}^{exp} &= -3.79 \pm 1.37 \pm 0.67 \% \text{ MIT - Caltech}^2 \\ A_{qe}^{exp} &= -2.60 \pm 0.90 \pm 0.46 \% \text{ MIT - Harward}^3 \\ A^{th} &= -3.43 \% \\ A_p^{th} &= -1.30 \% \end{aligned}$$

It should be pointed out that our numerical results, as shown in Fig. 3, are only slightly different from the ones obtained in Ref.[6], where a spin-dependent Faddeev spectral function has been used.

In Fig. 4a and 4b the results based upon prescription I are compared with the our previous one⁵, based upon prescription II (the corresponding explicit expressions for $G_{1(2)}^A$ are given in Ref.[5] and coincide, as already mentioned, with Eqs.(22) and (23) with the term \mathcal{L} dropped out). The results of the comparison (cf. Fig. 4a and 4b) show that at $\theta^* \approx 90^\circ$ the approximate method yields results very different from the correct one, whereas at $\theta^* \approx 0^\circ$ such a difference is not present. This is due to the fact that in the procedure II $R_{TL'}$ results to be proportional to the component of \vec{R} along \hat{q} , instead of being proportional to its transverse part, as it should be⁹, therefore it is affected by the difference between V^σ and R^σ , arising from the term proportional to q^σ which is present in the latter. For $\theta^* \approx 0^\circ$ the differences, as shown in Fig.4b, are very small over the whole range of the energy transfer considered, since $R_{T'}$ is unaffected by the extra term in R^σ (we recall that $A \propto R_{T'}$ only at the top of the qe peak, and the mixing with $R_{TL'}$ explains the small differences on the wings of the asymmetry).

From the above comparisons it turns out that the difference between the two procedures is almost entirely due to the proton contribution; for such a reason the correctness of our conclusion, reached in [5] using prescription II, about the possibility of obtaining information on the neutron form factors by properly minimizing the proton contribution are not affected by the use of prescription I. This will be illustrated in the next Section.

5. Minimizing the proton contribution

As shown in Figs. 1 and 2, the proton contribution to the asymmetry, corresponding to the polarization angle of the actual experiments is sizeable. Following our previous paper⁵, in this Section the possibility to minimize or even to make vanishing the proton contribution will be investigated. To this end we have analyzed the proton contribution to the asymmetry at the top of the qe peak, for different values of β , different values of the energy of the incident electron and different models for the proton form factors. The results are presented in Fig. 5. It can be seen that the

Fig. 1. The asymmetry corresponding to $\epsilon_1 = 574 \text{ MeV}$ and $\theta_e = 44^\circ$, vs. the energy transfer ν calculated by Eqs. (22) and (23) (solid line) and using the spin-dependent spectral function of Ref.[5]; the dotted (dashed) line represents the neutron (proton) contribution. The nucleon form factors of Ref. [10] have been used and the experimental data are from Ref.[2]. The arrow indicates the position of the qe peak.

Fig. 2 The same as in Fig. 2, but for $\theta_e = 51.1^\circ$. The experimental point, has been obtained , Ref.[2], after averaging over a 103 MeV interval around the qe peak, as explained in the text.

Fig. 3. Comparison of the asymmetry ($\epsilon_1 = 574 \text{ MeV}$ and $\theta_e = 44^\circ$) shown in Fig. 1 (calculated by Eqs. (22) and (23)) (solid line) with the one of Ref.[6] (dashed line), based on a Faddeev spin-dependent spectral function. The nucleon form factors of Ref. [11] have been used and the experimental data are from Ref.[2]. The arrow indicates the position of the qe peak.

Fig. 4a. The asymmetry and the neutron contribution for $\epsilon_1 = 574 \text{ MeV}$ and $\theta_e = 44^\circ$ vs. the energy transfer ν calculated using prescriptions I and II. Solid (dotted) line: the asymmetry (neutron contribution) corresponding to prescription I (Eqs. (22) and (23)); dashed (dot-dashed) line: the asymmetry (neutron contribution) corresponding to prescription II (Eqs. (22) and (23) without the term \mathcal{L}). The form factors of Ref. [10] have been used and the experimental data are from Ref.[2]. The arrow indicates the position of qe peak.

Fig. 4b. The same as in Fig. 4a, but for $\theta_e = 51.1^\circ$.

Fig. 5. The proton contribution to the asymmetry , at the top of the qe peak , vs. β , for $\theta_e = 75^\circ$. Solid line: $\epsilon_1 = 500 \text{ MeV}$, long-dashed line: $\epsilon_1 = 1000 \text{ MeV}$, short-dashed line: $\epsilon_1 = 1500 \text{ MeV}$, dotted line: $\epsilon_1 = 2000 \text{ MeV}$. The nucleon form factors of Ref. [10] have been used.

Fig. 6. The total asymmetry at the top of the qe peak, vs. Q^2 , for $\theta_e = 75^\circ$ and $\beta = 95^\circ$, using Eqs. (22) and (23). Solid line : Gari-Kruempelmann form factors¹⁰; dashed line: Blatnik-Zovko form factors¹³, dotted line: Hoehler et al form factors¹². The curves in the lower part of the figure represent the corresponding proton contributions.

Fig. 7. The same as in Fig. 6, but for the Galster form factors [11].

proton contribution is almost vanishing around $\beta \equiv \beta_c = 95^\circ$, in a large spectrum of values of incident electron energy; such a feature, moreover, weakly depends upon the model for the nucleon form factors. In Fig. 6, the asymmetry, and the proton contribution, vs of Q^2 , at fixed values of $\beta_c = 95^\circ$ and $\theta_e = 75^\circ$ is presented for three different models of the nucleon form factors (Refs.[10, 12-13]). It can be seen that the asymmetry is very sensitive to the neutron form factors, but from Fig. 6 it is not possible to assess whether the differences in the asymmetry are given by the differences in G_E^n or in G_M^n , since both of them vary within the models we have considered. In order to make our analysis a more stringent one, we have repeated the calculation by using the Galster model of the nucleon form factors¹¹, since within such a model G_E^n can be changed independently of G_M^n . In fact one has

$$\begin{aligned} G_M^n &= \mu_n G_E^p \\ G_E^n &= \frac{-\tau \mu_n}{(1 + \eta \tau)} G_E^p \end{aligned} \quad (35)$$

where $G_E^p = 1/(1 + Q^2/B)^2$, $B = 0.71(GeV/c)^2$ and η is a parameter. The resulting

asymmetry and proton contribution are shown in Fig. 7 for different values of η . Fig.7 illustrates how the total asymmetry can depend upon G_E^n , having a vanishing proton contribution.

It should be stressed that the proposed kinematics, which minimizes the proton contribution, corresponds to the qe peak, where the final state interaction is expected to play a minor role.

6. Summary and conclusion

The qe spin-dependent structure functions for a nucleus with $J = 1/2$ have been obtained by a proper procedure, based on the replacement of the exact hadronic tensor with its PWIA version. Our formal results are in agreement with the ones of Ref.[6], whereas the numerical calculations only slightly differ, which demonstrates the equivalence of the spin dependent spectral functions used in Ref.[5] and Ref.[6].

The origin of the differences between the predictions of the correct procedure and the ones^{4,5} based upon the replacement of the hadronic pseudovector $V^{\sigma 4}$ with its PWIA version R^σ , Eq.(6), have been clarified. In particular it has been shown that these differences are produced by the presence of an extra term proportional to the momentum transfer q^σ in the four-vector R^σ , Eq.(13). This extra term affects only the response function $R_{TL'}$.

Our analysis of the asymmetry, based on the correct expression of G_1^A and G_2^A given by Eqs.(22) and (23) respectively, has fully confirmed the main conclusions of our previous paper⁵, concerning : i) the relevance of the proton contribution for the experimental kinematics considered till now, and ii) the possibility of selecting a polarization angle, which leads at qe peak to an almost vanishing proton contribution for a wide range of the kinematical variable; within such a kinematical condition, the sensitivity of the asymmetry to the electric neutron form factor has been thoroughly investigated.

Calculations of the final state effects are in progress.

6. References

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